Name: _____ **Date:** _____

PSTAT 5A: Midterm 2 Review Problems

Spring 2023, with *Ethan P. Marzban*

MORE PROBLEMS WILL BE POSTED TO THE COURSE WEBSITE SHORTLY.

1. The Intelligence Quotient (IQ) has historically been used as a measure of a person's reasoning ability. Studies have shown that IQs are normally distributed with mean 100 and standard deviation 15.
   (a) If a person has been selected at random, what is the probability that their IQ is lower than 90?
   (b) According to the Wechsler Intelligence Scale, an IQ of between 120 and 129 is classified as "Superior". What proportion of the population have "Superior" IQs?
   (c) Mensa is an organization open only to those with extremely high IQs. The minimum IQ one must possess in order to be eligible for Mensa membership is 130- what proportion of the population qualify for Mensa memberships?
   (d) Caoimhe knows that 68% of the population have an IQ lower than hers. What IQ does Caoimhe have?

2. Around 1 out of every 3000 calico cats is born male. Suppose a sample of one thousand calico cats is taken (with replacement), and the number of male cats is recorded.
   (a) Define the random variable of interest, and call it X.
   (b) Show that X follows the Binomial Distribution, and identify its parameters.
   (c) What is the probability that the sample contains at least 2 male cats? (You will need to use Python to compute this.)
   (d) What is the expected number of male cats in the sample?
   (e) What is the standard deviation of the number of male cats in the sample?

3. Steel rods manufactured at a particular plant are meant to be 100 meters in length. Due to minor errors in the production cycle, rod lengths are actually uniformly distributed between 90 and 110 meters.
   (a) What is the probability that a randomly selected rod is between 97 and 103 meters in length?
   (b) What is the probability that a randomly selected rod is longer than 100 meters?
   (c) If a sample of 120 rods is to be taken (with replacement) and the number of rods over 100 meters in length is recorded, what is the probability that this sample contains exactly 60 rods that are longer than 100 meters?

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4. A shopowner believes that 22% of his customers prefer to pay with cash. To test this belief, he takes a representative sample of 150 customers and finds that 20 of these sampled customers prefer to pay with cash. **Use a two-sided alternative, and an $\alpha = 0.05$ level of significance**.

   (a) Write down the null and alternative hypotheses for this test. Be sure to use mathematical notation, and define any variables you use.

   (b) Compute the value of the test statistic.

   (c) What is the distribution of the test statistic, assuming the null hypothesis is correct? Be sure to check any relevant conditions!

   (d) Find the critical value of the test.

   (e) Now, conduct the hypothesis test and state your conclusions in the context of the problem.

5. Mary would like to determine the true average speed with which cars drive on a particular stretch of Highway 101. As such, she takes a representative sample of 40 cars; the average speed of these cars is 65mph and the standard deviation of the speeds of these 40 cars is 10mph.

   (a) Define the random variable of interest, $\overline{X}$

   (b) What distribution would we use to construct a 95% confidence interval for the true average speed of cars along this stretch of Highway 101? Be sure to include any/all relevant parameter(s), and check any/all conditions!

   (c) Construct a 95% confidence interval for the true average speed of cars along this stretch of Highway 101, and interpret your interval.

6. Consider a random variable X with the following p.m.f. (probability mass function):

k	-0.1	0.1	0.2	1
$\mathbb{P}(X = k)$	0.1	0.1	a	0.1

   (a) Find the value of a.

   (b) Compute $\mathbb{P}(\{X = 0\} \cup \{X = 1\})$.

   (c) Compute $\mathbb{P}(X \leq 0.5)$.

   (d) Find $\mathbb{E}[X]$.

   (e) Find $\text{SD}(X)$.

