YELLOW VERSION **Score:** ________ / **50**

PSTAT 5A / **MIDTERM EXAM 1** / Spring 2023 Instructor: **Ethan Marzban**

Name:____ **UCSB NetID:**____________________

First, then Last *NOT your Perm Number!*

Circle the section you attend:

Yuan 10 - 10:50am Jason 11 - 11:50am Nickolas 12 - 12:50pm Nickolas 1 - 1:50pm

Your Seat Number:____________

Person Sitting to your Left:____

Person Sitting to your Right:____

Instructions:

- You will have **65 minutes** to complete this exam.
- You are allowed the use of a single **8.5 × 11-inch** sheet, front and back, of notes. You are also permitted the use of **calculators**; the use of any and all other electronic devices (laptops, cell phones, airpods/headphones, etc.) is prohibited.
- For Multiple Choice Questions: fill in the bubble corresponding to your answer directly on the exam. Partial credit will **not** be awarded.
- For Free Response Questions: be sure to include **all** of your work! Correct answers with no supporting work will **not** receive full points.
- **PLEASE DO NOT DETACH ANY PAGES FROM THIS EXAM.**
- Good Luck!!!

Honor Code: In signing my name below, I certify that all work appearing on this exam is entirely my own and not copied from any external source. I further certify that I have not received any unauthorized aid while taking this exam.

× ____

Problem 1. True or False: If $\{x_i\}_{i=1}^n$ is a set of numbers with mean $\overline{x}$, then the mean of the set $\{ax_i\}_{i=1}^n$ for a fixed constant a is simply $a \cdot \overline{x}$. [1pts.]

- √ **True**
- ◯ False
- ◯ Not Enough Information to Determine

Problem 2. Events A and B are such that $\mathbb{P}(A) = 0.3$, $\mathbb{P}(B) = 0.8$, and $\mathbb{P}(A \cap B) = 0.24$. Select the statement that is correct. [1pts.]

- √ **A and B are independent, but not disjoint**
- ◯ A and B are disjoint, but not independent
- ◯ A and B are both disjoint and independent
- ◯ A and B are neither disjoint nor independent

Problem 3. Jana has run the following code: [1pts.]

```
def f(x, y):
    """return the sum of x and y"""
    x + y
```

What will be the output of running `f(1, 2)`?

- ◯ `3`
- √ **Nothing**
- ◯ An Error
- ◯ `[1, 2]`
- ◯ None of the above.

Problem 4. Suppose a password for a particular website must be 5 characters long, consisting of exactly 2 digits (0 through 9), 2 letters (A through Z), and 1 special character (!, @, #, $, %), <u>in that order</u>. What is the total number of passwords that can be constructed using this scheme? [1pts.]

- ◯ 1,300
- ◯ 73,125
- ◯ 292,500
- √ 338,000
- ◯ None of the above.

Problem 5. In a variable re-assignment statement in Python, which side of the equality does Python evaluate first? [1pts.]

- √ **Right**
- ◯ Left

Problem 6. Which of the following is **not** a measure of spread? [1pts.]

- ◯ Interquartile Range
- ◯ Standard Deviation
- √ **50th Percentile**
- ◯ Range
- ◯ None of the above

Problem 7. If the variable X contains measurements on the duration (in minutes) of 100 different flights from SBA to EWR, what is the correct classification of X? [1pts.]

- ◯ discrete
- √ **continuous**
- ◯ nominal
- ◯ ordinal

Problem 8. In order for $\mathbb{P}(A \mid B)$ to be defined for two events A and B, which of the following conditions must be true? **Select only ONE answer choice**. [1pts.]

- ◯ $\mathbb{P}(A) \neq 0$
- √ $\mathbb{P}(B) \neq 0$
- ◯ $\mathbb{P}(A \cap B) \neq 0$
- ◯ $\mathbb{P}(A \cup B) \neq 0$
- ◯ None of the above.

Problem 9. Guadalupe would like to visualize the relationship between a person's favorite color and their height. Which type of graph should she use? [1pts.]

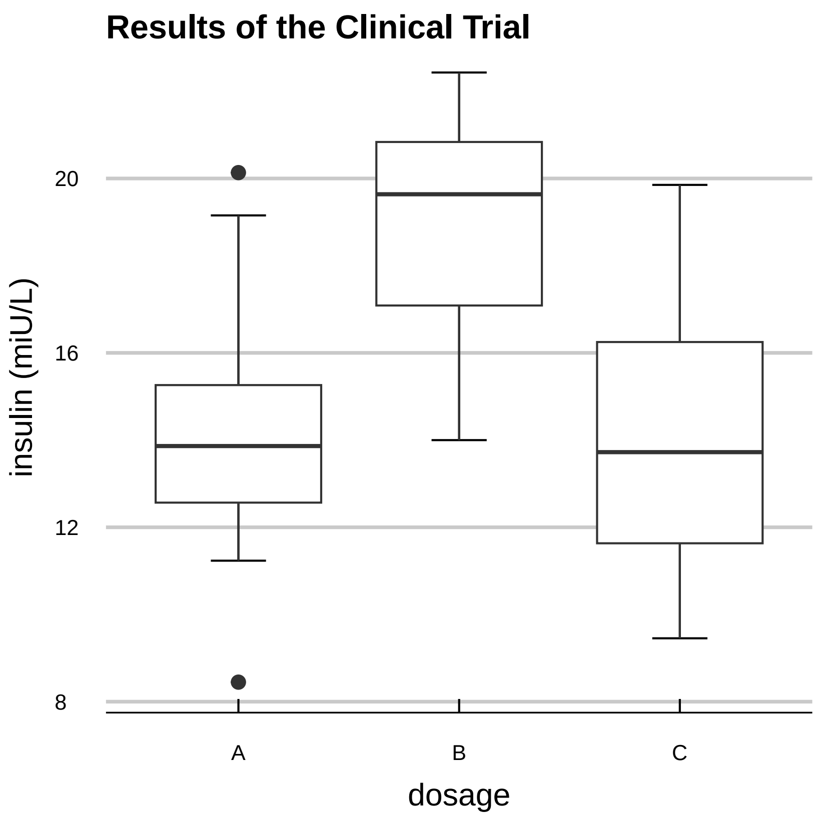
- ◯ A bargraph
- ◯ A histogram
- √ **A side-by-side boxplot**
- ◯ A scatterplot
- ◯ None of the above

Problem 10. In what module is the function `make_array()` found? [1pts.]

- √ `datascience`
- ◯ `numpy`
- ◯ `python_arrays`
- ◯ None of the above

Free Response Questions

Problem 11. In a clinical trial, subjects were administered one of three different dosages of a particular drug. 3 hours later, the insulin count (in miU/Liter) of each subject was taken and recorded. The results of the trial are displayed below:



(a) Provide the 5-number summary for the insulin levels of subjects who were administered dosage A. Round your numbers to the nearest decimal place. [3pts.]

Solution:

min	Q_1	median	Q_3	max
8.5	12.5	14.0	15.3	20.1

(These are, of course, only approximate.)

(b) Approximately what percent of subjects who were administered dosage C had insulin levels lower than 16.1 miU/L? [2pts.]

Solution: It appears that 16.1 is the third quartile of insulin measurements of individuals administered dosage C. As such, by definition of the third quartile, this means that approximately 75% of subjects administered dosage C has insulin levels lower than 16.1.

(c) Does there appear to be a difference in insulin levels across dosages? Explain in one or two brief sentences. [3pts.]

Solution: **Answers may vary**. Based on the relative positions of the boxplots, it seems that there was no significant difference in average insulin levels between individuals administered dosages A and C, whereas individuals administered dosage B appear to have (on average) higher insulin levels.

Problem 12. Consider the set of numbers

$$B = \{-2,\ -1.5,\ 0,\ 8\}$$

(a) Compute $\overline{b}$, the mean of B. [3pts.]

Solution:

$$\overline{b} = \frac{1}{4}[(-2) + (-1.5) + (0) + (8)] = \frac{4.5}{4} = \boxed{1.125}$$

(b) Compute the standard deviation of B. [4pts.]

Solution:

$$\begin{aligned} s_b^2 &= \frac{1}{5-1}\left[(-2-1.125)^2 + (-1.5-1.125)^2 + (0-1.125)^2 + (8-1.125)^2\right] \\ &= \frac{1}{4}(65.1875) \approx 16.297 \\ s_b &= \sqrt{s_b^2} = \sqrt{16.297} \approx \boxed{4.037} \end{aligned}$$

(c) Compute the median of B. [2pts.]

Solution:

$$B = \{\cancel{-2},\ -1.5,\ 0,\ \cancel{8}\} \implies \text{median}(B) = \frac{-1.5+0}{2} = \boxed{-0.75}$$

Problem 13. It is known that 5% of people in the town of *Gauchoville* are affected by a particular disease. There is a test for this disease, however it is imperfect- specifically, it has a 25% false positive rate and a 10% false negative rate.

(a) Define appropriate notation (i.e. define relevant events), and translate the information provided into the problem to be in terms of the events you define. [2pts.]

> **Solution:** Let D denote the event "a randomly selected person has the disease", and $+$ denote "a person tests positive". From the problem statement, we therefore have
>
> $$\mathbb{P}(D) = 0.05; \quad \mathbb{P}(+ \mid D^{\complement}) = 0.25; \quad \mathbb{P}(- \mid D) = 0.1$$

(b) What is the probability that a randomly selected person will both have the disease <u>and</u> test positive? [2pts.]

> **Solution:** We seek $\mathbb{P}(D \cap +)$. By the Multiplication Rule,
>
> $$\mathbb{P}(D \cap +) = \mathbb{P}(+ \mid D) \cdot \mathbb{P}(D) = (1 - 0.1) \cdot (0.05) = \boxed{0.045 = 4.5\%}$$

(c) What is the probability that a randomly selected person will test positive? [3pts.]

> **Solution:** We seek $\mathbb{P}(+)$, which can be computed using the Law of Total Probability:
>
> $$\begin{aligned} \mathbb{P}(+) &= \mathbb{P}(+ \mid D) \cdot \mathbb{P}(D) + \mathbb{P}(+ \mid D^{\complement}) \cdot \mathbb{P}(D^{\complement}) \\ &= (1 - 0.1) \cdot (0.05) + (0.25) \cdot (1 - 0.05) \\ &= (0.9) \cdot (0.05) + (0.25) \cdot (0.95) = \boxed{0.2825 = 28.25\%} \end{aligned}$$

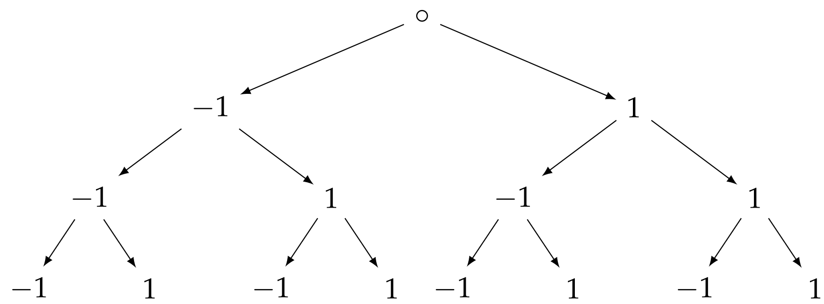
(d) Suppose Fatima has tested herself for the disease, and her test returned a positive result. What is the probability that she actually has the disease? [3pts.]

> **Solution:** We seek $\mathbb{P}(D \mid +)$, which can be computed using Bayes' Rule:
>
> $$\mathbb{P}(D \mid +) = \frac{\mathbb{P}(+ \mid D) \cdot \mathbb{P}(D)}{\mathbb{P}(+)} = \frac{0.045}{0.2825} = \boxed{\frac{18}{113} \approx 15.9\%}$$

Problem 14. Three numbers are to be selected from the set $\{-1, 1\}$. Assume we replace the numbers after each draw, and assume that the order in which the numbers are selected is important.

(a) Write down the outcome space Ω for this experiment. [3pts.]

Solution: Using a Tree will be easiest:



Alternatively, we could have listed out all of the elements explicitly:

$$\Omega = \{(-1,-1,-1),\ (-1,-1,\ 1),\ (-1,\ 1,-1),\ (1,-1,-1),\\ (-1,\ 1,\ 1),\ (1,-1,\ 1),\ (1,\ 1,-1),\ (1,\ 1,\ 1)\}$$

(b) How many elements are in Ω? [2pts.]

Solution: From part (a), we see there are 8 outcomes in Ω. We could have also found this using a slot diagram with three slots (one for each number):

$$\underline{2} \times \underline{2} \times \underline{2} = 8$$

(c) Are we justified in using the Classical Approach to probability in this problem? Why or why not? [1pts.]

Solution: No, since it is not stated that the numbers were selected "at random". However, I decided to award everyone the full point on this problem (across versions) so long as they wrote *something*.

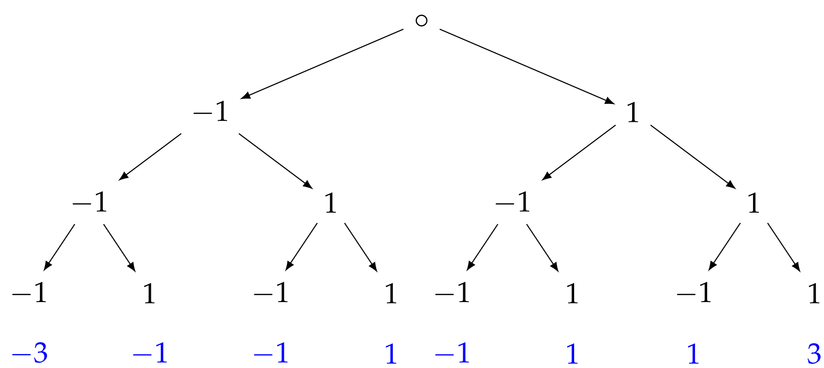
(d) Let A denote the event "the first number selected was greater than the second number selected." Write down the mathematical formulation of A; i.e. identify the outcomes that are contained in A. [3pts.]

Solution: If the first number selected was -1, then the second number must have been 1 if it is to be greater than the first number selected. If first number selected was 1, then there are no possibilities for the second number; as such,

$$A = \{(-1, 1, -1), (-1, 1, 1)\}$$

(e) Let E denote the event "the sum of the three numbers selected is 1". Compute $\mathbb{P}(E)$ using the classical approach to probability. [4pts.]

Solution: We should figure out what sum each of the 8 outcomes in Ω correspond to:



We can now see that there are 3 outcomes in which the sum of the three numbers is 1:

$$E = \{(-1, 1, 1), (1, -1, 1), (1, 1, -1)\}$$

and so, by the Classical Approach to Probability,

$$\mathbb{P}(E) = \frac{3}{8} = 37.5\%$$

Please Note: For full credit, you needed to have justified your answer for the numerator somehow, either by writing the mathematical formulation of E or by making some sort of explicit counting argument. If you just jumped straight to $\#(E) = 3$, you did not receive full credit.

You may use this page for scratch work, if necessary. Keep in mind that NOTHING on this page will be graded.

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