Homework 3

PSTAT 5A: Spring 2023, with Ethan P. Marzban

Instructions

- Please submit your work to Gradescope by no later than **11:59pm on MONDAY, APRIL 24**. As a reminder, late homework will not be accepted.
- Recall that you will be asked to upload a **single** PDF containing your work for *both* the programming and non-programming questions to Gradescope.
  - You can merge PDF files using either Adobe Acrobat, or using adobe's online PDF merger at this link.

Problem 1: Independence

Suppose we have two events E and F such that $\mathbb{P}(F \mid E) = \mathbb{P}(F)$. Since $\mathbb{P}(F)$ represents our beliefs on the event F and $\mathbb{P}(F \mid E)$ represents our beliefs on the event F after observing the event E, this equality asserts that our beliefs about F remain unchanged in the presence of E. This is the key notion of **independence**:

Definition: Independence

Two events E and F are said to be independent if $\mathbb{P}(F \mid E) = \mathbb{P}(F)$, which in turn implies $\mathbb{P}(E \mid F) = \mathbb{P}(E)$ [as you will show in part (a)].

So, the colloquial interpretation of two events being independent of each other is that "one does not affect the other".

a. If E and F are events such that $\mathbb{P}(F \mid E) = \mathbb{P}(F)$, show that this implies $\mathbb{P}(E \mid F) = \mathbb{P}(E)$. **Hint:** Use Bayes' Rule.

b. If E and F are independent events, show that $\mathbb{P}(E \cap F) = \mathbb{P}(E) \cdot \mathbb{P}(F)$. **Hint:** Use the multiplication rule. **As an aside:** It turns out that $\mathbb{P}(E \cap F) = \mathbb{P}(E) \cdot \mathbb{P}(F)$ **if and only if** E and F are independent; in other words, you **CANNOT** always compute the probability of an intersection as a product of marginal (i.e. unconditional) probabilities.

c. For the following pairs of events E and F, determine whether they are independent or not, or if there is insufficient information. Justify your answers using the definition of independence, or the results you derived in parts (a) and (b) above.
   i. $\mathbb{P}(E) = 0.5,\ \mathbb{P}(F) = 0.5,\ \mathbb{P}(E \cap F) = 0.1$
   ii. $\mathbb{P}(F) = 0.5,\ \mathbb{P}(F \mid E) = 0.5$
   iii. $\mathbb{P}(F) = 0.5,\ \mathbb{P}(E) = 0.4$

Problem 2: Motherboards

Motherboards in *GauchoVille* are manufactured in two factories, named A and B. It is known that 63% of all motherboards are manufactured in factory A, and the remaining 37% are manufactured in factory B. Of the motherboards produced in factory A 3% are defective; of the motherboards produced in factory B 5% are defective. A motherboard is selected at random.

a. What is the probability that the motherboard will be defective?
b. If the motherboard is defective, what is the probability that it was manufactured in factory B?

Problem 3: Trees

A recent ecological survey recorded the species (Maple, Oak, and Pine) of several trees and the type of soil on which they were found (Clay, Loamy, Sandy). The results of the survey are summarized in the following contingency table:

```
       Soil
Tree    Clay Soil Loamy Soil Sandy Soil
  Maple         5         25         15
  Oak          15         20         10
  Pine         10          5         20
```

a. What is the correct classification (discrete, continuous, ordinal, nominal) of `Tree`?
b. What is the correct classification (discrete, continuous, ordinal, nominal) of `Soil`?
c. What is the correct visualization for `Soil Type`?
d. If a tree is to be selected at random from the trees that were included in the survey, what is the probability that it is a Pine tree?
e. If a tree is to be selected at random from the trees that were included in the survey, what is the probability that it was found on Loamy Soil?
f. A tree is selected at random from the trees included in the survey, and it is noted to be an Oak tree. What is the probability that it was found on sandy soil?

Problem 4: Programming

Part (a): Write a function called `abs_diff()` that takes in two inputs `x` and `y` and returns $|x - y|$. Test your function on at least three different inputs.

Part (b): Write a function called `is_odd()` that takes in a single input `x` and returns `True` if `x` is odd and `False` if `x` is even. Make sure that `is_odd(3)` returns `True` and `is_odd(0)` returns `False`. **Hint:** On Lab03, how did we *mathematically* check whether a number is even or not?