Name:______________________________ **Date:**________________

PSTAT 5A: Discussion Worksheet 02

Spring 2023, with *Ethan P. Marzban*

1. Consider two events A and B. Express the following events using **only** these two events, along with unions, intersections, and complements. Additionally, for each part, sketch a Venn Diagram of the specified event.

   (a) Both A and B occur.

   > **Solution:** $A \cap B$

   (b) Either A or B occur.

   > **Solution:** $A \cup B$

   (c) Neither A nor B occur.

   > **Solution:** $A^\complement \cap B^\complement$; or, equivalently, $(A \cup B)^\complement$

   (d) Either A or B occur, but not both.

   > **Solution:** $(A \cap B^\complement) \cup (A^\complement \cap B)$ (i.e. either A happens and B does not, or B happens but A does not).

2. Consider the experiment of tossing a fair coin and rolling a fair 6-sided die (and recording the outcome of both the coin flip and the die roll).

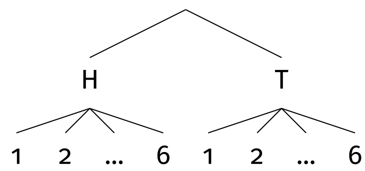
   (a) Use a table to express the outcome space of this experiment.

   > **Solution:**

coin \ *die*	**1**	**2**	**3**	**4**	**5**	**6**
H	$(H, 1)$	$(H, 2)$	$(H, 3)$	$(H, 4)$	$(H, 5)$	$(H, 6)$
T	$(T, 1)$	$(T, 2)$	$(T, 3)$	$(T, 4)$	$(T, 5)$	$(T, 6)$

(b) Use a tree diagram to express the outcome space of this experiment.

Solution:



(c) Find the probability that the die lands on an even number.

Solution: Let E denote the event "the die lands on an even number." Because the die is fair, we can utilize the classical approach to probability. As such, we need only to find the number of outcomes in which the die lands on an even number. We see this to be 6; since there are 12 outcomes in Ω, we have

$$\mathbb{P}(E) = \frac{6}{12} = \frac{1}{2} = 50\%$$

(d) Find the probability that the coin lands on 'heads'.

Solution: Let F denote the event "the coin lands on 'heads"'. There are 6 outcomes in which the coin lands 'heads', meaning, by the classical approach to probability,

$$\mathbb{P}(F) = \frac{6}{12} = \frac{1}{2} = 50\%$$

(e) Find the probability that the die lands on an even number, or the coin lands 'heads'.

Solution: Letting the events E and F be defined as in the previous parts, we seek $\mathbb{P}(E \cup F)$. By the **Addition Rule**, we have

$$\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F)$$

We computed $\mathbb{P}(E)$ and $\mathbb{P}(F)$ in parts (a) and (b) above; all that remains is to compute $\mathbb{P}(E \cap F)$. Note that the event $E \cap F$ corresponds to "the die landed on an even number and the coin landed heads". Therefore,

$$E \cap F = \{(H, 2),\ (H, 4),\ (H, 6)\}$$

meaning

$$\mathbb{P}(E \cap F) = \frac{3}{12} = \frac{1}{4}$$

and so

$$\mathbb{P}(E \cup F) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4} = 75\%$$

3. A jar contains 3 red candies, 4 blue candies, and 2 purple candies. Three candies are to be drawn at random, and their color is to be recorded. The order in which the colors appear is not important.

(a) How many elements are in the outcome space Ω associated with this experiment?

Solution: Because selection is made without regard to the order of colors, we use choosing factors (as opposed to ordering factors). That is, we recall (from Lecture03) that the number of ways to select k objects from a total of n (not replacing the objects in between draws) is $\binom{n}{k}$. Here, $n = 4+3+2 = 9$ and $k = 3$, so the number of outcomes in Ω is

$$\begin{aligned}\binom{9}{2} &= \frac{9!}{3! \cdot (9-3)!} = \frac{9!}{3! \cdot 6!} = \frac{9 \times 8 \times 7 \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{(3 \times 2 \times 1) \cdot (\cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1})} \\ &= \frac{9 \times 8 \times 7}{6} = 3 \times 4 \times 7 = 84\end{aligned}$$

(b) In how many outcomes do we observe exactly 3 red candies?

Solution: There is only 1 configuration in which all three red candies are selected. The other way to think about this is: the number of ways to select 3 red candies from a total of 3 red candies is

$$\binom{3}{3} = \frac{3!}{3! \cdot (3-3)!} = \frac{3!}{3! \cdot 0!} = \frac{3 \times 2 \times 1}{3 \times 2 \times 1} = 1$$

4. Consider an outcome space $\Omega = \{a, b, c\}$ for arbitrary elements a, b, and c. Suppose that $\mathbb{P}(\{a\}) = \mathbb{P}(\{b\})$, and that $\mathbb{P}(\{c\}) = 0.1$. What is $\mathbb{P}(\{a\})$? **Hint:** Recall the second axiom of probability; i.e. that $\mathbb{P}(\Omega) = 1$.

Solution: Since $\Omega = \{a, b, c\}$, we have

$$\mathbb{P}(\Omega) = \mathbb{P}(\{a\}) + \mathbb{P}(\{b\}) + \mathbb{P}(\{c\})$$

Let p denote $\mathbb{P}(\{a\})$; we then have that $\mathbb{P}(\{b\}) = p$ as well. Additionally, $\mathbb{P}(\{c\}) = 0.1$, meaning

$$\mathbb{P}(\Omega) = p + p + 0.1 = 2p + 0.1$$

As the hint suggests, we know that this must equal 1. As such,

$$2p + 0.1 = 1 \implies 2p = 0.9 \implies \boxed{p = 0.45 = 45\%}$$