**Name:**______________________________ **Date:**________________

# PSTAT 5A: Discussion Worksheet 04

*Spring 2023*, with *Ethan P. Marzban*

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**1.** In a particular textbook, it is found that each page contains a typo with probability 10%, independently across pages. Suppose 120 pages are selected (assume selection is done with replacement), and the number of pages that have typos is recorded.

(a) Define the random variable of interest, and call it $X$.

(b) Does $X$ follow the Binomial distribution? If so, state its parameters.

(c) What is the probability that the textbook contains exactly 17 pages with typos?

(d) What is the probability that the textbook contains between 16 and 18 (inclusive on both ends) pages with typos?

(e) What is the expected number of pages in the textbook that contain typos?

(f) What is the variance of the number of pages in the textbook that contain typos?

**2.** Consider the following game: a fair six-sided die is rolled. If the number showing is 1 or 2, you win a dollar; if the number showing is 3, 4, or 5 you win 2 dollars; if the number showing is 6, you lose 1 dollar. Let $W$ denote your net winnings after playing this game once.

(a) Write down the state space $S_W$ of $W$.

(b) Find the p.m.f. of $W$.

(c) What are your expected winnings after one round of the game?

**3.** Let $X$ be a random variable with the following p.m.f. (probability mass function):

| $k$ | $-3$ | $-2$ | $0.5$ | $3$ | $4$ |
|---|---|---|---|---|---|
| $\mathbb{P}(X = k)$ | 0.12 | 0.08 | 0.43 | $a$ | 0.27 |

(a) Find the value of $a$.

(b) Compute $\mathbb{P}(X = 1)$.

(c) Compute $\mathbb{P}(X \leq 0.5)$.

(d) Compute $\mathbb{P}(X \leq 1)$.

(e) Compute $\mathbb{E}[X]$.

(f) Compute $\mathrm{SD}(X)$.

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**4.** (**The Geometric Distribution**) Consider the following situation: suppose we toss a $p$−coin repeatedly, and we let $X$ denote the number of tosses (including our final toss) until we observe our first heads. So, for example, the outcome

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would correspond to a value of $X = 4$. If we assume independence across trials, then $X$ is said to follow the **Geometric Distribution with parameter** $\boldsymbol{p}$, notated $X \sim \text{Geom}(p)$.

(a) What is the state space of $X$? **Hint:** is there an upper limit to the values $X$ can attain? What about a lower limit?

(b) The event $\{X = k\}$ means “we observed our first heads on the $k^{\text{th}}$ toss”. Find $\mathbb{P}(X = k)$.

(c) It turns out that $\mathbb{E}[X] = 1/p$. If we toss a fair coin, what is the expected number of tosses (including the final toss) needed to observe our first heads?

 