# PSTAT 5A: Lecture 10 Exercise 2 *Spring 2023: with Ethan P. Marzban*

**Exercise 2**

At a certain company, it is known that 65% of employees are from underrepresented minorities (UMs). A representative sample of 80 employees is taken, and the proportion of people from UMs is recorded.

a. Define the random variable of interest.
b. What is the probability that greater than than 50% of people in the sample are from UMs?
c. What is the probability that the proportion of people in the sample who are from UMs lies within 5% of the true proportion of 65%?

### Part (a)

We define $\widehat{P}$ to be the proportion of people in a sample of 80 employees that are from underrepresented minorities.

### Part (b)

We seek the quantity $\mathbb{P}(\widehat{P} \geq 0.5)$. It would be nice if we were able to invoke the Central Limit Theorem for Proportions; to do so, however, we need to check the success-failure conditions.

1) $np = (80)(0.65) = 52 \geq 10$
2) $n(1-p) = 28 \geq 10$

Both conditions are met, so we know that

$$\widehat{P} \sim \mathcal{N}\left(0.65,\ \sqrt{\frac{(0.65)(1-0.65)}{80}}\right) \sim \mathcal{N}(0.65,\ 0.0533)$$

Therefore,

$$\begin{aligned} \mathbb{P}(\widehat{P} \geq 0.5) &= 1 - \mathbb{P}(\widehat{P} < 0.5) \\ &= 1 - \mathbb{P}\left(\frac{\widehat{P} - 0.65}{0.0533} < \frac{0.5 - 0.63}{0.0533}\right) = 1 - \mathbb{P}\left(\frac{\widehat{P} - 0.65}{0.0533} < -2.44\right) = 1 - 0.0073 = \boxed{99.27\%} \end{aligned}$$

where we have used standardization to note that $[(\widehat{P} - 0.63)/0.0533] \sim \mathcal{N}(0,\ 1)$ and used a standard normal table to look up the left-area corresponding to a $z-$score of $-2.44$.

### Part (c)

We now seek $\mathbb{P}(0.6 \leq \widehat{P} \leq 0.7)$, which we first write as

$$\mathbb{P}(\widehat{P} \leq 0.7) - \mathbb{P}(\widehat{P} \leq 0.6)$$

Standardizing yields

$$\begin{aligned}\mathbb{P}(\widehat{P} \le 0.7) - \mathbb{P}(\widehat{P} \le 0.6) &= \mathbb{P}\left(\frac{\widehat{P} - 0.65}{0.0533} \le \frac{0.7 - 0.65}{0.0533}\right) - \mathbb{P}\left(\frac{\widehat{P} - 0.65}{0.0533} \le \frac{0.6 - 0.65}{0.0533}\right) \\ &= \mathbb{P}\left(\frac{\widehat{P} - 0.65}{0.0533} \le 0.94\right) - \mathbb{P}\left(\frac{\widehat{P} - 0.65}{0.0533} \le -0.94\right) \\ &= 0.8264 - 0.1736 = \boxed{65.28\%}\end{aligned}$$