Name:____ **Date:**____________________

PSTAT 5A: Homework 02

Summer Session A 2023, with *Ethan P. Marzban*

As a reminder, homework is neither collected nor graded. We encourage you to stop by Office Hours to ask any questions you may have about your work, or the problems themselves!

1. A recent survey at a cinema revealed that 80% of moviegoers purchase popcorn and 60% purchase a drink. Additionally, 62.5% of those who purchase popcorn also purchase a drink.

(a) Define events, and translate the information provided in the problem. Remember: the events you define should not be conditional.

> **Solution:** Let P denote the event "a randomly selected moviegoer purchases popcorn" and D denote "a randomly selected moviegoer purchases a drink." Then, the problem tells us that
>
> $$\mathbb{P}(P) = 0.8; \quad \mathbb{P}(D) = 0.6; \quad \mathbb{P}(D \mid P) = 0.625$$

(b) What is the probability that a randomly selected moviegoer purchases both popcorn and a drink?

> **Solution:** We seek $\mathbb{P}(P \cap D)$, which we compute using the **Multiplication Rule:**
>
> $$\mathbb{P}(P \cap D) = \mathbb{P}(D \mid P) \cdot \mathbb{P}(P) = (0.625) \cdot (0.8) = \boxed{0.5}$$

(c) What is the probability that a randomly selected moviegoer purchases neither popcorn nor a drink?

> **Solution:** We seek $\mathbb{P}(P^C \cap D^C)$. As we have seen previously, this is computed using
>
> $$\begin{aligned} \mathbb{P}(P^C \cap D^C) &= 1 - \mathbb{P}(P \cup D) \\ &= 1 - [\mathbb{P}(P) + \mathbb{P}(D) - \mathbb{P}(P \cap D)] \\ &= 1 - (0.8 + 0.6 - 0.5) = \boxed{0.1} \end{aligned}$$

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2. Consider the experiment of selecting a number at random from the set of positive integers between 1 and 100, inclusive on both ends, and recording the number selected.

(a) Write down the outcome space Ω for this experiment.

Solution: Since the only outcomes that could result from this experiment are the positive integers between 1 and 100, inclusive, we have

$$\Omega = \{1, 2, \cdots, 100\}$$

(b) What is the probability that the number selected is even?

Solution: Let A denote the event "the number selected was even." There are 50 even integers between 1 and 100, inclusive, meaning there are 50 elements in A and so desired probability is simply

$$\mathbb{P}(A) = \frac{50}{100} = \frac{1}{2} = 50\%$$

(c) What is the probability that the number selected is strictly greater than 65?

Solution: Let B denote the event "the number selected was strictly greater than 65" There are 35 integers greater than 65 but less than or equal to 100, meaning there are 35 elements in B and so desired probability is simply

$$\mathbb{P}(B) = \frac{35}{100} = \frac{7}{20} = 35\%$$

(d) What is the probability that the number selected is even, given that it is strictly greater than 65?

Solution: Let C denote the event "the number selected is even" and let B be defined as in part (c) above. We seek $\mathbb{P}(C \mid B)$, which we compute using

$$\mathbb{P}(C \mid B) = \frac{\mathbb{P}(C \cap B)}{\mathbb{P}(B)} = \frac{\#(C \cap B)}{\#(B)}$$

There are 18 numbers between 66 and 100, inclusive, that are even; hence $\#(C \cap B) = 18$ and

$$\mathbb{P}(C \mid B) = \frac{18}{35} \approx 51.43\%$$

(e) If the number is a multiple of three, what is the probability that it is odd?

Solution: Let D denote the event "the number selected is a multiple of three" and E denote the event "the number selected is odd". Similar to part (d) above, we have

$$\mathbb{P}(E \mid D) = \frac{\#(E \cap D)}{\#(D)}$$

There are 16 numbers that are odd multiples of three between 1 and 100, inclusive, meaning $\#(E \cap D) = 16$; additionally, there are 33 multiples of three in Ω meaning $\#(D) = 33$ and so

$$\mathbb{P}(E \mid D) = \frac{16}{33} = 48.\overline{48}\%$$

3. A researcher is interested in the relationship between exercise habits and mental health. To that effect, she surveyed several individuals on their exercise habits as well as their mental health; the results of her survey are displayed in the following contingency table:

```
                   Mental_Health
Exercise_Habits      Poor Fair Good
  Sedentary            30   25   20
  Moderately Active    40   35   30
  Very Active          45   50   25
```

A person is selected at random. Use the Classical Approach to Probability wherever necessary.

(a) What is the probability that the selected person has a sedentary lifestyle?

Solution: Let S denote the event "the person has a sedentary lifestyle". Then, by the Classical Approach to Probability,

$$\mathbb{P}(S) = \frac{\#(S)}{\#(\Omega)} = \frac{30 + 25 + 20}{30 + 25 + 20 + 40 + 35 + 30 + 45 + 50 + 25} = \frac{75}{300}$$

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(b) What is the probability that the selected person has 'fair' mental health?

Solution: Let F denote "the person has 'fair' mental health"; then

$$\mathbb{P}(F) = \frac{\#(F)}{\#(\Omega)} = \frac{25 + 35 + 50}{30 + 25 + 20 + 40 + 35 + 30 + 45 + 50 + 25} = \frac{110}{300}$$

(c) What is the probability that the selected person has both a 'moderately active' lifestyle and 'good' mental health?

Solution: Let M denote "the person has a moderately active lifestyle" and G denote "the person has 'good' mental health". Then

$$\mathbb{P}(M \cap G) = \frac{30}{30 + 25 + 20 + 40 + 35 + 30 + 45 + 50 + 25} = \frac{30}{300}$$

(d) Given that the person has 'good' mental health, what is the probability that they have a 'very active' lifestyle?

Solution: Let G be defined as above, and let V denote "the selected person has a very active lifestyle." Then

$$\mathbb{P}(V \mid G) = \frac{\#(V \cap G)}{\#(G)} = \frac{25}{20 + 30 + 25} = \frac{25}{75}$$

(e) If the person has a 'moderately active' lifestyle, what is the probability that they have 'fair' mental health?

Solution: Let M and F be defined as above. Then

$$\mathbb{P}(F \mid M) = \frac{\#(F \cap M)}{\#(M)} = \frac{35}{40 + 35 + 30} = \frac{35}{105}$$

4. Consider events E and F with $\mathbb{P}(E) = 0.5$, $\mathbb{P}(F) = 0.7$, and $\mathbb{P}(E \cap F) = 0.35$.

(a) What is $\mathbb{P}(E \cup F)$?

Solution: By the **Addition Rule**,

$$\mathbb{P}(E \cup F) = \mathbb{P}(E) + \mathbb{P}(F) - \mathbb{P}(E \cap F) = 0.5 + 0.7 - 0.35 = 0.85$$

(b) What is $\mathbb{P}(E \mid F)$?

Solution: By the definition of conditional probability,

$$\mathbb{P}(E \mid F) = \frac{\mathbb{P}(E \cap F)}{\mathbb{P}(F)} = \frac{0.35}{0.70} = \frac{1}{2}$$

(c) What is $\mathbb{P}(F \mid E)$?

Solution: By the definition of conditional probability,

$$\mathbb{P}(F \mid E) = \frac{\mathbb{P}(E \cap F)}{\mathbb{P}(E)} = \frac{0.35}{0.50} = \frac{7}{10}$$

(d) Are E and F mutually exclusive? Why or why not?

Solution: Definitionally, events E and F are mutually exclusive if $E \cap F = \varnothing$ which means $\mathbb{P}(E \cap F) = 0$. Here, however, $\mathbb{P}(E \cap F) = 0.35 \neq 0$ meaning the two events are not mutually exclusive.

(e) Are E and F independent? Why or why not?

Solution: We could use any of the three conditions for independence to note that E and F are indeed independent; for example, we could note that

$$\mathbb{P}(E \cap F) = 0.35 = (0.5) \cdot (0.7) = \mathbb{P}(E) \cdot \mathbb{P}(F)$$

The other conditions also hold.

