



PSTAT 5A: Homework 03

Summer Session A 2023, with Ethan P. Marzban

Please make sure to periodically work on these problems throughout the week, so that you are well-prepared for the quiz on Friday!

1. Consider the random variable X with the following probability mass function (p.m.f.):

k	-2.4	-0.2	0	4
$\mathbb{P}(X = k)$	0.1	0.4	a	0.2

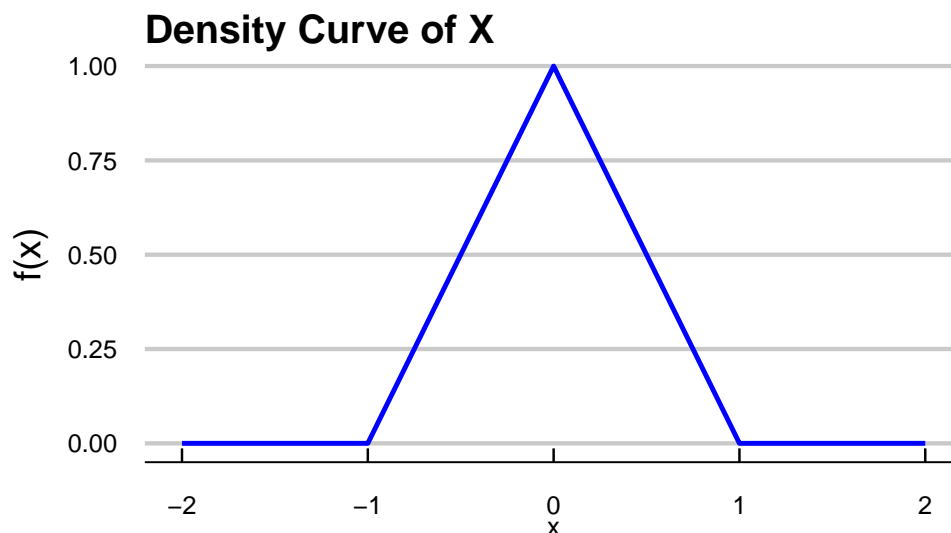
where a is an as-of-yet unknown constant.

- (a) What is the value of a ?
 - (b) What is $\mathbb{P}(-2.4 \leq X < -0.2)$?
 - (c) What is $\mathbb{P}(X \geq 0)$?
 - (d) If $F_X(x)$ denotes the cumulative distribution function (c.d.f.) of X at x , what is the value of $F_X(0)$?
 - (e) What is $\mathbb{E}[X]$?
 - (f) What is $\text{Var}(X)$?
2. In a large parking lot, 35% of cars are electric vehicles. A random sample of 200 cars is taken with replacement from this lot, and the number of electric vehicles in the sample is recorded.
- (a) Define the random variable of interest and call it X .
 - (b) Identify the distribution of X , taking care to list out any/all parameter(s)! Also, be sure to check and relevant conditions.
 - (c) What is the probability that this sample contains exactly 72 electric vehicles?
 - (d) What is the probability that between 70 and 73 of the cars in this sample (inclusive on both ends) are electric vehicles?
 - (e) What is the expected number of electric vehicles we expect to observe in this sample?
 - (f) What is the standard deviation of the number of electric vehicles we expect to observe in this sample?
3. The weight of a randomly-selected fish from *Lake Gaucho* (in bounds) is normally distributed with mean 4 lbs and standard deviation 1.3 lbs. A fish is selected at random from *Lake Gaucho* and its weight is recorded.

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- (a) Define the random variable of interest.
 - (b) What is the probability that this fish weighs less than 2 lbs?
 - (c) What is the probability that this fish weighs more than 4.8 lbs?
 - (d) What is the probability that this fish weighs between 2.5 lbs and 3.8 lbs?
 - (e) Suppose, now, that a random sample of 10 fish is caught (assume that the weights of fish in *Lake Gaucho* are independent), and the number of fish that weight between 2.5 and 3.8 lbs is recorded. What is the probability that exactly 3 of these fish weigh between 2.5 and 3.8 lbs?
Hint: you will need to define another random variable.
4. The duration of a flight from SBA to SEA is uniformly distributed between 150 mins and 170 mins. A flight from SBA to SEA is selected at random, and its duration is recorded.
- (a) Define the random variable of interest.
 - (b) What is the expected duration of a randomly-selected flight from SBA to SEA?
 - (c) What is the standard deviation of the duration of a randomly-selected flight from SBA to SEA?
 - (d) What is the probability that a randomly-selected flight from SBA to SEA will have a duration of 162 mins?
 - (e) What is the probability that a randomly-selected flight from SBA to SEA will have a duration of between 162 and 165 mins?
 - (f) What is the probability that a randomly-selected flight from SBA to SEA will have a duration of between 162 and 180 mins?
5. A random variable X has the following density curve:



- (a) What is the state space of X ?



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- (b) Verify that $f_X(X)$ is a valid probability density function (p.d.f.) **Hint:** Recall that there are two properties a function $f_X(X)$ must satisfy in order to be a valid p.d.f..
- (c) What is $\mathbb{P}(X \leq -1)$?
- (d) What is $\mathbb{P}(X \geq 0)$?
- (e) What is $\mathbb{P}(-1 \leq X \leq -0.5)$?
- (f) What is $\mathbb{P}(-1 < X < -0.5)$?
- (g) What is $\mathbb{P}(0.25 \leq X \leq 0.75)$?
6. In each of the parts below, determine whether the provided quantity is a population parameter or a sample statistic. Use this to further determine whether the quantity is a deterministic (i.e. nonrandom) constant, or a random variable.
- (a) The proportion of trees in the Amazon rainforest that are mango trees.
- (b) The average AQI (Air Quality Index) of 25 randomly-selected cities on August 1, 2022.
- (c) The longest amount of time a human can hold their breath underwater.
- (d) The median number of words of a sample of 100 randomly-selected books.
7. Suppose that 33% of a particular country's population has a college degree. A representative sample of 243 people is taken, and the proportion of these people who have a college degree is recorded.
- (a) Define the parameter of interest, and use the notation discussed in Lecture.
- (b) Define the random variable of interest; again use proper notation.
- (c) Check whether the success-failure conditions are satisfied.
- (d) What is the probability that over 30% of the sample have college degrees?
- (e) What is the probability that the proportion of people in the sample who have college degrees lies within 5% of the true proportion of 33%?
8. The U.S. Department of Housing and Urban Development defines a person or household to be "rent-burdened" if 30% or more of the individual/household's income is spent on housing. A recent survey revealed that 42% of households in a representative sample of 150 households were rent-burdened.
- (a) Define the parameter of interest.
- (b) Define the random variable of interest.
- (c) Construct a 95% confidence interval for the true proportion of rent-burdened households, and interpret your interval in the context of the problem.
- (d) Would you expect an 80% confidence interval for the true proportion of rent-burdened households to be wider or narrower than the 95% confidence interval you constructed in part (c)? Explain briefly.



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- (e) Construct an 80% confidence interval for the true proportion of rent-burdened households, and interpret your interval in the context of the problem.