Lecture 12, Exercise 3

> Exercise 3
>
> As a film critic, you are interested in determining the true proportion of people that have watched *The Mandalorian.* You take a representative sample of 100 people, and note that 47% of these people have watched *The Mandalorian.*
>
> a) Construct a 95% confidence interval for the proportion of people that have watched *The Mandalorian*, and interpret your interval in the context of the problem.
>
> b) When constructing an 85% confidence interval for the proportion of people that have watched *The Mandalorian*, would you expect this interval to be wider or shorter than the interval you found in part (a)?
>
> c) Now, actually construct an 85% confidence interval for the proportion of people that have watched *The Mandalorian* and see if this agrees with your answer to part (b).

Solutions:

As always, we start by defining notation:

- p = The proportion of all people who have watched *The Mandalorian*
- $\widehat{P}$ = The proportion of a representative sample of size 100 who have watched *The Mandalorian*

Additionally, let $\hat{p} = 0.47$ denote the proportion of the already-taken sample of 100 people who have watched *The Mandalorian.*

Part (a)

We should first verify that we are able to invoke the Central Limit Theorem for Proportions (otherwise, we don't know what distribution to use when finding the confidence coefficient!) Since we don't have access to the true value of p, we instead use the Substitution Approximation:

1) $n\hat{p} = (100)(0.47) = 47 \geq 10$

2) $n(1-\hat{p}) = 100(0.53) = 53 \geq 10$

So, we are able to invoke the CLTP to conclude that

$$\widehat{P} \sim \mathcal{N}\left(p,\ \sqrt{\frac{p(1-p)}{n}}\right)$$

meaning we use the normal distribution to find the confidence coefficient. We have already seen that, in the case of a normally-dsitributed sample proportion, a confidence level of 95% corresponds to a confidence coefficient of 1.96; hence, our Confidence Interval is

$$\hat{p} \pm 1.96 \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (0.47) \pm 1.96 \cdot \sqrt{\frac{(0.47)\cdot(1-0.47)}{100}} = \boxed{[0.3722\ ,\ 0.5678]}$$

One possible interpretation of this interval is:

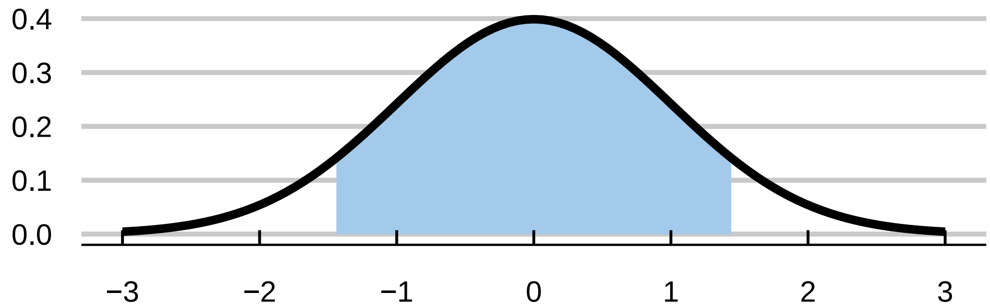
> We are 95% confident that the true proportion of people who have watched *The Mandalorian* is between 37.22% and 56.78%.

Part (b)

We previously saw that smaller confidence levels corespond to narrower confidence intervals; since 85% is smaller than 95%, we would expect a 85% confidence interval for p (as defined above) to be **narrower** than our 95% confidence interval from part (a).

Part (c)

The only thing that changes between part (a) and this part is our confidence coefficient. Now, we pick z^* to be the value such that the following area is 85%:



This means that the tails each have probability $(1-0.85)/2-0.075$, meaning z^* can be found as either negative one times the 7.5^{th} percentile of the standard normal distribution, or as the $[1-(0.075)] \times 100 = 92.5^{\text{th}}$ percentile. Either way we find $z^* = 1.44$, and so our confidence interval becomes

$$\hat{p} \pm 1.44 \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = (0.47) \pm 1.44 \cdot \sqrt{\frac{(0.47) \cdot (1-0.47)}{100}} = \boxed{[0.3981\ ,\ 0.5419]}$$

which has interpretation

> We are 85% confident that the true proportion of people who have watched *The Mandalorian* is between 39.81% and 54.19%.

As a quick sanity check, note that this interval is in fact narrower than our interval from part (a)!