PSTAT 5A: Quiz 01

Summer Session A 2023, with *Ethan P. Marzban*

Please Note: There were several versions of the quiz, each of them with slightly different contexts and numbers.

1. Consider the list of numbers

$$X = \{-1,\ 0,\ 3,\ 5\}$$

(a) (3 points) Compute $\overline{x}$, the mean of X.

Solution:

$$\begin{aligned}\overline{x} &= \frac{1}{n}\sum_{i=1}^{n} x_i \\ &= \frac{1}{4}(-1+0+3+5) = \frac{7}{4} = 1.75\end{aligned}$$

(b) (5 points) Compute s_X, the standard deviation of X. Express your final answer as a fraction.

Solution: We first find the variance:

$$\begin{aligned}s_X^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\overline{x})^2 \\ &= \frac{1}{4-1}\cdot\left[\left(-1-\frac{7}{4}\right)^2+\left(0-\frac{7}{4}\right)^2+\left(3-\frac{7}{4}\right)^2+\left(5-\frac{7}{4}\right)^2\right] = \frac{91}{12}\end{aligned}$$

Hence, we have

$$s_X = \sqrt{s_X^2} = \sqrt{\frac{91}{12}} = \frac{\sqrt{273}}{6} \approx 2.754$$

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2. At *Pickles and Swiss*, it is found that 60% of customers order pickles on their sandwiches, 50% order Swiss Cheese on their sandwiches, and 30% order both pickles and Swiss Cheese on their sandwiches. A customer is to be selected at random.

Solution: Let A denote the event "the customer orders pickles", and let B denote the event "the customer orders Swiss Cheese." Then, from the problem statement, we have

$$\mathbb{P}(A) = 0.6; \quad \mathbb{P}(B) = 0.5; \quad \mathbb{P}(A \cap B) = 0.3$$

By The Way: This is what I mean by "defining your events in words".

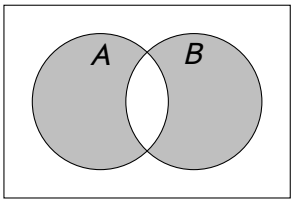
(a) (4 points) What is the probability that they order either pickles or Swiss Cheese (or both) on their sandwich?

Solution: We seek $\mathbb{P}(A \cup B)$, which can be computed using the **Addition Rule**:

$$\begin{aligned}\mathbb{P}(A \cup B) &= \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) \\ &= 0.6 + 0.5 - 0.3 = \boxed{0.8}\end{aligned}$$

(b) (5 points) What is the probability that they order either pickles or Swiss Cheese but not both on their sandwich? You should sketch a Venn Diagram for full points.

Solution: We seek $\mathbb{P}[(A \cap B^C) \cup (A^C \cap B)]$. To find this probability, we sketch a **Venn Diagram**:



From this, we see that

$$\begin{aligned}\mathbb{P}[(A \cap B^C) \cup (A^C \cap B)] &= \mathbb{P}(A) + \mathbb{P}(B) - 2 \cdot \mathbb{P}(A \cap B) \\ &= 0.6 + 0.5 - 2 \cdot 0.3 = \boxed{0.5}\end{aligned}$$

(c) (4 points) What is the probability that they order neither pickles nor Swiss Cheese on their sandwich?

Solution: We seek $\mathbb{P}(A^C \cap B^C)$. By **DeMogran's Laws**, we know that

$$(A^C \cap B^C) = A \cup B$$

As such, by the **Complement Rule**,

$$\mathbb{P}(A^C \cap B^C) = 1 - \mathbb{P}(A \cup B) = 1 - (0.8) = 0.2$$

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