



PSTAT 5A: Quiz 03

Summer Session A 2023, with Ethan P. Marzban

WRITE YOUR ANSWERS ON A SEPARATE SHEET OF PAPER; DO NOT TRY TO WRITE THEM ON THIS PDF

1. Emily believes that there is no difference between the average height of a poplar tree and the average height of an aspen tree. To test her beliefs, she takes a representative sample of 27 poplar trees and another representative sample of 31 aspen trees, and records the heights (in feet) of these trees. The results of her data are summarized below:

	Sample Mean	Sample Std. Dev.
Poplar	82.1	6.21
Aspen	78.6	10.44

Assume all independence and normality conditions hold; additionally, let “Population 1” refer to the set of all poplar trees and let “Population 2” refer to the set of all aspen trees. Furthermore, use a two-sided alternative and a 5% level of significance wherever necessary.

Note: You may need to use Python for certain parts of this question. If you do use Python, clearly write down the command you used.

- (a) (1 point) Define the parameters of interest, μ_1 and μ_2 .

Solution: Let μ_1 denote the average height (in feet) of all poplar trees and let μ_2 denote the average height (in feet) of all aspen trees.

- (b) (2 points) State the null and alternative hypotheses.

Solution:

$$\begin{bmatrix} H_0 : \mu_1 = \mu_2 \\ H_A : \mu_1 \neq \mu_2 \end{bmatrix} = \begin{bmatrix} H_0 : \mu_2 - \mu_1 = 0 \\ H_A : \mu_2 - \mu_1 \neq 0 \end{bmatrix}$$

- (c) (2 points) Compute the observed value of the test statistic.

Solution:

$$ts = \frac{(\bar{y} - \bar{x})}{\sqrt{\frac{s_x^2}{n_1} + \frac{s_y^2}{n_2}}} = \frac{(78.6 - 82.1)}{\sqrt{\frac{6.21^2}{27} + \frac{10.44^2}{31}}} = -1.57$$

- (d) (4 points) Assuming the null is true, what distribution does the test statistic follow? Be sure to include any/all relevant parameter(s)!

Solution: Since we are told to assume all independence and normality conditions hold, we know that, under the null, the test statistic will be distributed according to a t -distribution with degrees of freedom given by the Satterthwaite Approximation:

$$\begin{aligned} df &= \text{round} \left\{ \frac{\left[\left(\frac{s_x^2}{n_1} \right) + \left(\frac{s_y^2}{n_2} \right) \right]^2}{\frac{\left(\frac{s_x^2}{n_1} \right)^2}{n_1-1} + \frac{\left(\frac{s_y^2}{n_2} \right)^2}{n_2-1}} \right\} \\ &= \text{round} \left\{ \frac{\left[\left(\frac{6.21^2}{27} \right) + \left(\frac{10.44^2}{31} \right) \right]^2}{\frac{\left(\frac{6.21^2}{27} \right)^2}{27-1} + \frac{\left(\frac{10.44^2}{31} \right)^2}{31-1}} \right\} = \text{round} \{49.83554\} = 50 \end{aligned}$$

Hence, we have that

$$TS \stackrel{H_0}{\sim} t_{50}$$

- (e) (2 points) What is the critical value of the test?

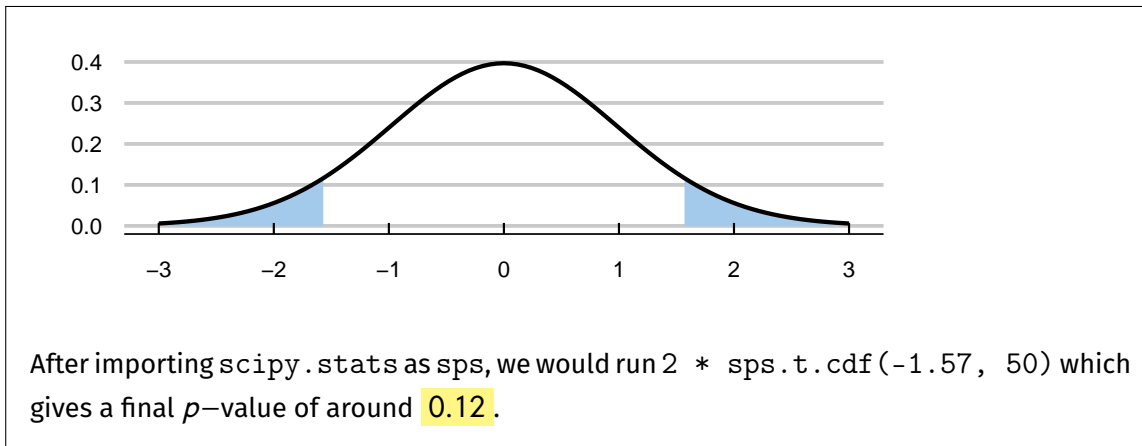
Solution: There are a couple of ways we could solve this. one would be to use Python, running the code

$$-\text{sps.t.ppf}(0.025, 50)$$

(after importing the `scipy.stats` module with the nickname `sps`); the other would be to use our t -table (since 50 degrees of freedom *does* appear on it). Either way we find a critical value of around **2.01**.

- (f) (2 points) What is the p -value of the observed value of the test statistic?

Solution: Here, we *must* use Python as our t -table does not directly give us areas. Since we are using a two-sided alternative, our p -value is the region



(g) (3 points) Conduct the test, and phrase your conclusions in the context of the problem.

Solution: Since we are conducting a two-sided test, we reject the null whenever:

- the absolute value of the test statistic exceeds the critical value
- the observed p -value is less than the level of significance

(recall that these are equivalent). Note that:

- $|-1.57| = 1.57 \not< 2.01$
- $0.12 \not< 0.05$

Therefore, we fail to reject the null:

At a 5% level of significance, there was insufficient evidence to reject the null that the average height of poplar and aspen trees are the same, in favor of the alternative that the average heights are not the same.

THE QUIZ CONTINUES ON THE NEXT PAGE

2. The results of an ANOVA are displayed in the following ANOVA table:

	DF	Sum Sq.	Mean Sq.	F-value	P(> F)
Btw. Groups	7	14	2	<Blank1>	0.4574
Residuals	22	44	2		

(a) (1 point) How many observations were there in total?

Solution: Recall that the denominator degrees of freedom (which appear in the second entry of the “DF” column) is computed using $n - k$ where n denotes the total number of observations and k denotes the number of groups. Additionally, the numerator degrees of freedom (which appear in the first entry of the “DF” column) are computed using $k - 1$; since $7 = k - 1$ this means we had $k = 8$ groups and hence

$$22 = n - 8 \implies n = 30$$

(b) (2 points) What should go in <Blank1>? Be sure to show your work!

Solution: The F -statistic is computed as the ratio of MS_G and the MS_E :

$$F = \frac{MS_G}{MS_E} = \frac{2}{2} = 1$$

(c) (2 points) Let μ_1, \dots, μ_k denote the k group means. What are the null and alternative hypotheses that were tested when computing the p -value appearing in the ANOVA table above? (You should clearly specify what the value of k was in this problem, and you should also be sure to use proper notation/phrasing.)

Solution: First note that we found $k = 8$ in part (a) above. In ANOVA, the null is always that the group means are all equal:

$$H_0 : \mu_1 = \dots = \mu_8$$

The alternative is that at least one group mean differs from the rest; hence

$$\left[\begin{array}{l} H_0 : \mu_1 = \dots = \mu_8 \\ H_A : \text{at least one group mean differs from the rest} \end{array} \right.$$