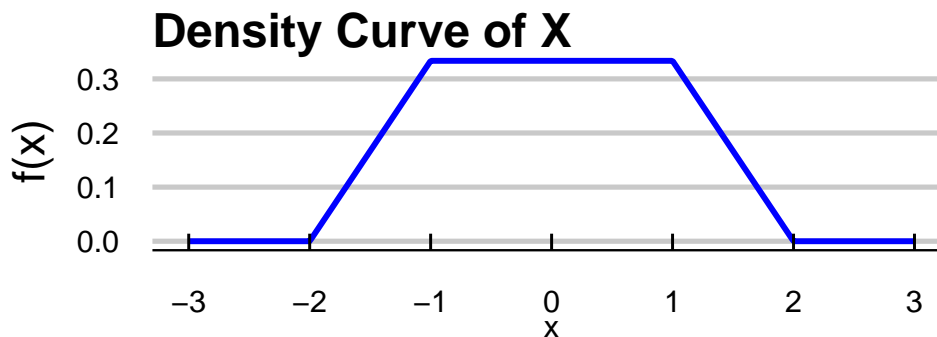




## PSTAT 5A: Discussion Worksheet 03

Summer Session A 2023, with Ethan P. Marzban

- Consider the following game: a fair six-sided die is rolled. If the number showing is '1' or '2', you win a dollar; if the number showing is '3', '4', or '5' you win 2 dollars; if the number showing is '6', you lose 1 dollar. Let  $W$  denote your net winnings after playing this game once.
  - Write down the state space  $S_W$  of  $W$ .
  - Find the p.m.f. of  $W$ . **Hint:** You may want to define a new variable to keep track of the result of the die roll.
  - What are your expected winnings after one round of the game?
- The random variable  $X$  has the following density curve (in case it is difficult to read,  $f_X(x)$  attains a constant value of  $1/3$  for  $-1 \leq x \leq 1$ ):



- What is  $S_X$ , the state space of  $X$ ?
  - Let  $F_X(x)$  denote the cumulative distribution function (c.d.f.) of  $X$  at  $x$ . What is the value of  $F_X(1)$ ?
  - What is  $\mathbb{P}(-1 \leq X \leq 1)$ ?
  - What is  $\mathbb{P}(1 \leq X \leq 3)$ ?
- The time a randomly-selected person will spend waiting in line for the new hit restaurant *Gau-choYum* varies according to a normal distribution with mean 15 minutes and standard deviation 4 minutes. Suppose a person is selected at random, and the amount of time (in minutes) they spend waiting in line is recorded.
    - Define the random variable of interest, and call it  $X$ .

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- (b) Using proper notation, write down the distribution of  $X$ .
- (c) What is the probability that this randomly-selected person will wait exactly 12 minutes in line?
- (d) What is the probability that this randomly-selected person will wait less than 16 minutes?
- (e) What is the probability that this randomly-selected person will wait between 12 and 17 minutes?
- (f) Suppose now that a representative sample of 10 people is taken with replacement, and the number of people (in this group of 10) that wait between 12 and 17 minutes is recorded. What is the probability that precisely 4 of these 10 people wait between 12 and 17 minutes? **Hint:** You may need to define a new random variable and identify its distribution; if you do so, be sure to check any/all relevant conditions!